Chapter 18, Solution 1.

$$f'(t) = \delta(t+2) - \delta(t+1) - \delta(t-1) + \delta(t-2)$$

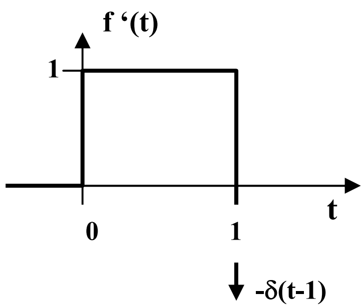
$$j\omega F(\omega) = e^{j2\omega} - e^{j\omega} - e^{-j\omega} + e^{-j\omega 2}$$

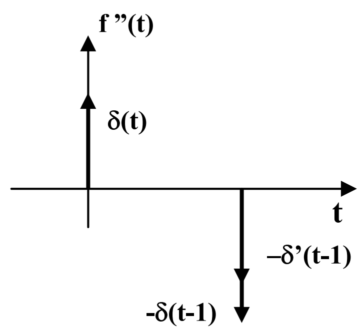
$$= 2\cos 2\omega - 2\cos\omega$$

$$F(\omega) = \underline{\mathbf{\frac{2[\cos 2\omega - \cos\omega]}{j\omega}}}$$

Chapter 18, Solution 2.

$$f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$$





$f''(t) = \delta(t) - \delta(t-1) - \delta'(t-1)$

Taking the Fourier transform gives

$$-\omega^2 F(\omega) = 1 - e^{-j\omega} - j\omega e^{-j\omega}$$

$$F(\omega) = \underline{\mathbf{\frac{(1 + j\omega)e^{j\omega} - 1}{\omega^2}}}$$

or $F(\omega) = \int_0^1 t\, e^{-j\omega t}\, dt$

But $\int x\, e^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1) + c$

$$F(\omega) = \frac{e^{-j\omega}}{(-j\omega)^2}(-j\omega t - 1)\Big|_0^1 = \underline{\mathbf{\frac{1}{\omega^2}\left[(1 + j\omega)e^{-j\omega} - 1\right]}}$$

Chapter 18, Solution 3.

$$f(t) = \frac{1}{2}t, -2 < t < 2, \qquad f'(t) = \frac{1}{2}, -2 < t < 2$$

$$F(\omega) = \int_{-2}^{2} \frac{1}{2} t\, e^{j\omega t} dt = \frac{e^{-j\omega t}}{2(-j\omega)^2}(-j\omega t - 1)\Big|_{-2}^{2}$$

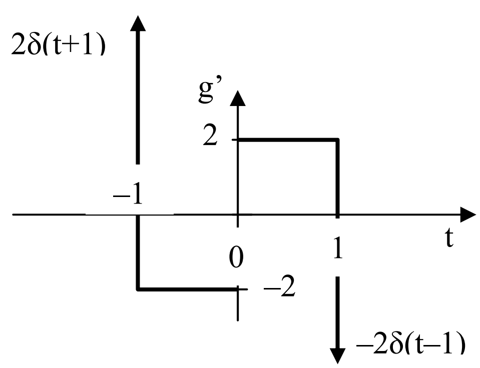
$$= -\frac{1}{2\omega^2}\left[e^{-j\omega 2}(-j\omega 2 - 1) - e^{j\omega 2}(j\omega 2 - 1)\right]$$

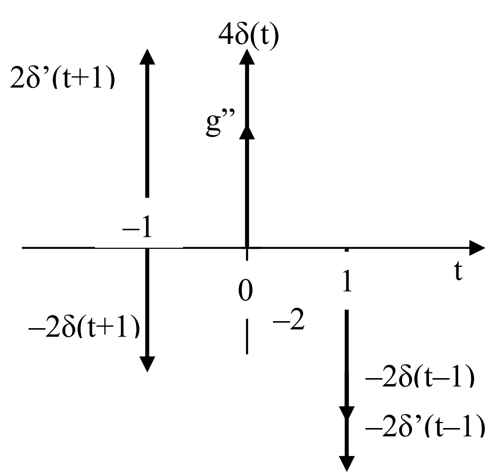
$$= -\frac{1}{2\omega^2}\left[- j\omega 2\left(e^{j\omega 2} + e^{j\omega 2}\right) + e^{j\omega 2} - e^{-j\omega 2}\right]$$

$$= -\frac{1}{2\omega^2}\left(- j\omega 4 \cos 2\omega + j2 \sin 2\omega\right)$$

$$F(\omega) = \underline{\boldsymbol{\frac{j}{\omega^2}(\sin 2\omega - 2\omega \cos 2\omega)}}$$

Chapter 18, Solution 4.



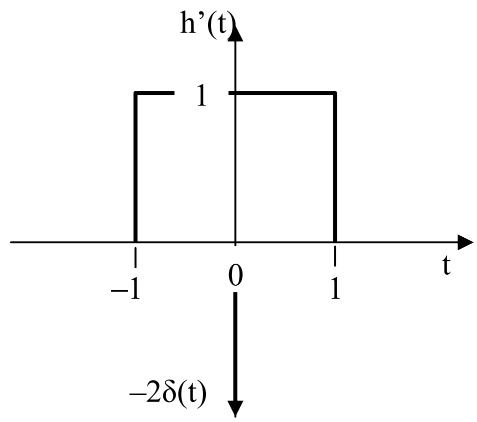


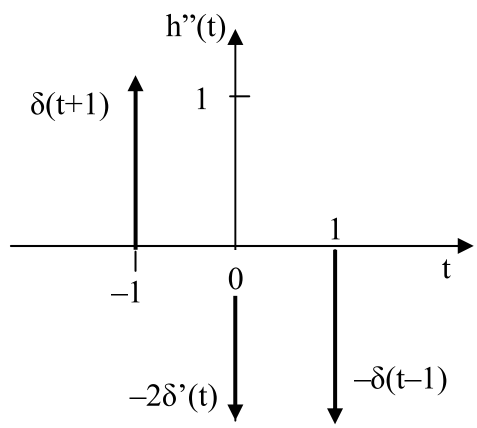
$$g'' = -2\delta(t+1) + 2\delta'(t+1) + 4\delta(t) - 2\delta(t-1) - 2\delta'(t-1)$$

$$(j\omega)^2 G(\omega) = -2e^{j\omega} + 2j\omega e^{j\omega} + 4 - 2e^{-j\omega} - 2j\omega e^{-j\omega}$$
$$= -4\cos\omega - 4\omega\sin\omega + 4$$

$$G(\omega) = \frac{4}{\omega^2}(\cos\omega + \omega\sin\omega - 1)$$

Chapter 18, Solution 5.





$$h''(t) = \delta(t+1) - \delta(t-1) - 2\delta'(t)$$

$$(j\omega)^2 H(\omega) = e^{j\omega} - e^{-j\omega} - 2j\omega = 2j\sin\omega - 2j\omega$$

$$H(\omega) = \frac{2j}{\omega} - \frac{2j}{\omega^2}\sin\omega$$

Chapter 18, Solution 6.

$$F(\omega) = \int_{-1}^{0} (-1)e^{-j\omega t}dt + \int_{0}^{1} te^{-j\omega t}dt$$

$$\text{Re}\,F(\omega) = -\int_{-1}^{0} \cos\omega t dt + \int_{0}^{1} t\cos\omega t dt$$

$$= -\frac{1}{\omega}\sin\omega t \,\Big|_{-1}^{0} + \left(\frac{1}{\omega^2}\cos\omega t + \frac{t}{\omega}\sin\omega t\right)\Big|_{0}^{1} = \underline{\frac{1}{\omega^2}(\cos\omega - 1)}$$

Chapter 18, Solution 7.

(a) f_1 is similar to the function f(t) in Fig. 17.6.

$$f_1(t) = f(t-1)$$

$$\text{Since } F(\omega) = \frac{2(\cos\omega - 1)}{j\omega}$$

$$F_1(\omega) = e^{j\omega}F(\omega) = \underline{\mathbf{\frac{2e^{-j\omega}(\cos\omega - 1)}{j\omega}}}$$

Alternatively,

$$f_1'(t) = \delta(t) - 2\delta(t-1) + \delta(t-2)$$

$$j\omega F_1(\omega) = 1 - 2e^{-j\omega} + e^{-j2\omega} = e^{-j\omega}(e^{j\omega} - 2 + e^{j\omega})$$

$$= e^{-j\omega}(2\cos\omega - 2)$$

$$F_1(\omega) = \underline{\mathbf{\frac{2e^{-j\omega}(\cos\omega - 1)}{j\omega}}}$$

(b) f_2 is similar to f(t) in Fig. 17.14.
$f_2(t) = 2f(t)$

$$F_2(\omega) = \underline{\mathbf{\frac{4(1 - \cos\omega)}{\omega^2}}}$$

$$= \frac{5e^{-j2t}}{2-j2} + \frac{5e^{+j2t}}{2+j2} = \frac{5}{2\sqrt{2}}\left[e^{-j(2t-45°)} + e^{j(2t-45°)}\right]$$

$$= \frac{5}{\sqrt{2}}\cos(2t-45°)$$

$$v_o(2) = \frac{5}{\sqrt{2}}\cos(4-45°) = \frac{5}{\sqrt{2}}\cos(229.18°-45°)$$

$v_o(2)$ = **-3.526 V**

Chapter 18, Solution 37.

$$2 \| j\omega = \frac{j2\omega}{2+j\omega}$$

By current division,

$$H(\omega) = \frac{I_o(\omega)}{I_s(\omega)} = \frac{\frac{j2\omega}{2+j\omega}}{4+\frac{j2\omega}{2+j\omega}} = \frac{j2\omega}{j2\omega+8+j4\omega}$$

$$H(\omega) = \mathbf{\frac{j\omega}{4+j3\omega}}$$

Chapter 18, Solution 38.

$$V_i(\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$V_o(\omega) = \frac{10}{10+j\omega 2}V_i(\omega) = \frac{5}{5+j\omega}\left(\pi\delta(\omega) + \frac{1}{j\omega}\right)$$

$$\text{Let } V_o(\omega) = V_1(\omega) + V_2(\omega) = \frac{5\pi\delta(\omega)}{5+j\omega} + \frac{5}{j\omega(5+j\omega)}$$

$$V_2(\omega) = \frac{5}{s(s+5)} = \frac{A}{s} + \frac{B}{s+5} \longrightarrow A = 1,\ B = -1, \quad s = j\omega$$

$$V_2(\omega) = \frac{1}{j\omega} - \frac{1}{5+j\omega} \longrightarrow v_2(t) = \frac{1}{2}\text{sgn}(t) - e^{-5t}$$

$$V_1 = \frac{5\pi\delta(\omega)}{5+j\omega} \longrightarrow v_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{5\pi\delta(\omega)}{5+j\omega} e^{j\omega t} d\omega$$

$$v_1(t) = \frac{5\pi}{2\pi} \cdot \frac{1}{5} = 0.5$$

$$v_0(t) = v_1(t) + v_2(t) = 0.5 + 0.5\,\text{sgn}(t) - e^{-5t}$$

But $\text{sgn}(t) = -1 + 2u(t)$

$$v_o(t) = +0.5 - 0.5 + u(t) - e^{-5t}u(t) = \underline{\mathbf{u(t) - e^{-5t}u(t)}}$$

Chapter 18, Solution 39.

$$V_s(\omega) = \int_{-\infty}^{\infty} (1-t)e^{-j\omega t} dt = \frac{1}{j\omega} + \frac{1}{\omega^2} - \frac{1}{\omega^2} e^{-j\omega}$$

$$I(\omega) = \frac{V_s(\omega)}{10^3 + j\omega x10^{-3}} = \underline{\frac{10^3}{10^6 + j\omega}\left(\frac{1}{j\omega} + \frac{1}{\omega^2} - \frac{1}{\omega^2} e^{-j\omega}\right)}$$

Chapter 18, Solution 40.

$$\ddot{v}(t) = \delta(t) - 2\delta(t-1) + \delta(t-2)$$
$$-\omega^2 V(\omega) = 1 - 2e^{-j\omega} + e^{j\omega 2}$$

$$V(\omega) = \frac{1 - 2e^{-j\omega} + e^{-j\omega 2}}{-\omega^2}$$

Now $$Z(\omega) = 2 + \frac{1}{j\omega} = \frac{1 + j2\omega}{j\omega}$$

$$I = \frac{V(\omega)}{Z(\omega)} = \frac{2e^{j\omega} - e^{j\omega 2} - 1}{\omega^2} \cdot \frac{j\omega}{1 + j2\omega}$$

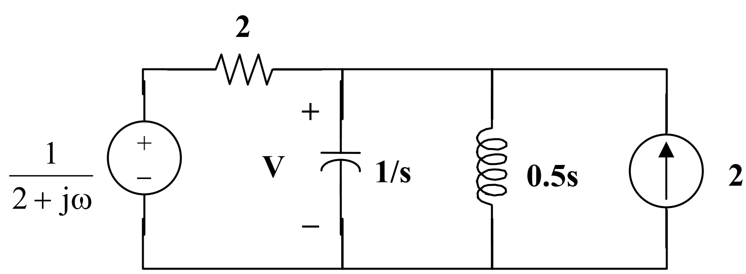
$$= \frac{1}{j\omega(0.5 + j\omega)}\left(0.5 + 0.5e^{-j\omega 2} - e^{-j\omega}\right)$$

But $\quad \dfrac{1}{s(s+0.5)} = \dfrac{A}{s} + \dfrac{B}{s+0.5} \longrightarrow A = 2, B = -2$

$$I(\omega) = \frac{2}{j\omega}\left(0.5 + 0.5e^{j\omega 2} - e^{-j\omega}\right) - \frac{2}{0.5 + j\omega}\left(0.5 + 0.5e^{-j\omega 2} - e^{-j\omega}\right)$$

$$i(t) = \underline{\mathbf{\frac{1}{2}sgn(t) + \frac{1}{2}sgn(t-2) - sgn(t-1) - e^{-0.5t}u(t) - e^{-0.5(t-2)}u(t-2) - 2e^{-0.5(t-1)}u(t-1)}}$$

Chapter 18, Solution 41.



$$\frac{V - \dfrac{1}{2 + j\omega}}{2} + j\omega V + \frac{2V}{j\omega} - 2 = 0$$

$$\left(j\omega - 2\omega^2 + 4\right)V = j4\omega + \frac{j\omega}{2 + j\omega} = \frac{-4\omega^2 + j9\omega}{2 + j\omega}$$

$$V(\omega) = \underline{\frac{2j\omega(4.5 + j2\omega)}{(2 + j\omega)(4 - 2\omega^2 + j\omega)}}$$

$i_o(t) \;=\; \underline{\mathbf{2\,sgn(t) - 2\,sgn(t-1) - 4e^{-2t}u(t) + 4e^{-2(t-1)}u(t-1)A}}$

Chapter 18, Solution 43.

$$20\,\text{mF} \;\longrightarrow\; \frac{1}{j\omega C} = \frac{1}{j20x10^{-3}\omega} = \frac{50}{j\omega}, \qquad i_s = 5e^{-t} \;\longrightarrow\; I_s = \frac{1}{5+j\omega}$$

$$V_o = \frac{40}{40+\dfrac{50}{j\omega}} I_s \bullet \frac{50}{j\omega} = \frac{50}{(s+1.25)(s+5)}, \qquad s = j\omega$$

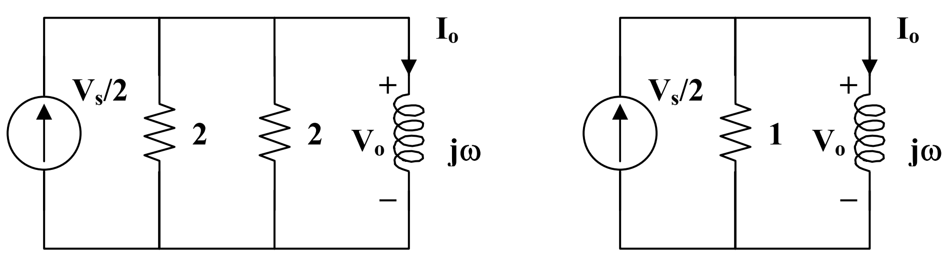
$$V_o = \frac{A}{s+1.25} + \frac{B}{s+5} = \frac{40}{3}\left[\frac{1}{j\omega+1.25} - \frac{1}{j\omega+5}\right]$$

$$\underline{v_o(t) = \frac{40}{3}(e^{-1.25t} - e^{-5t})u(t)}$$

Chapter 18, Solution 44.

$1\text{H} \longrightarrow j\omega$

We transform the voltage source to a current source as shown in Fig. (a) and then combine the two parallel 2Ω resistors, as shown in Fig. (b).



(a) (b)

$$2\|2 = 1\Omega, \;\; I_o = \frac{1}{1+j\omega}\cdot\frac{V_s}{2}$$

$$V_o = j\omega I_o = \frac{j\omega V_s}{2(1+j\omega)}$$

$$\ddot{v}_s(t) = 10\delta(t) - 10\delta(t-2)$$
$$j\omega V_s(\omega) = 10 - 10e^{-j2\omega}$$

$$V_s(\omega) = \frac{10\left(1 - e^{-j2\omega}\right)}{j\omega}$$

Hence $V_o = \dfrac{5\left(1 - e^{-j2\omega}\right)}{1 + j\omega} = \dfrac{5}{1 + j\omega} - \dfrac{5}{1 + j\omega}e^{-j2\omega}$

$$v_o(t) = 5e^{-t}u(t) - 5e^{-(t-2)}u(t-2)$$
$$v_o(1) = 5e^{-1} - 1 - 0 = \underline{\mathbf{1.839\ V}}$$

Chapter 18, Solution 45.

$$V_o = \frac{\frac{1}{j\omega}}{2 + j\omega + \frac{1}{j\omega}}(2) = \frac{2}{(j\omega + 1)^2} \longrightarrow \underline{v_o(t) = 2te^{-t}u(t)}$$

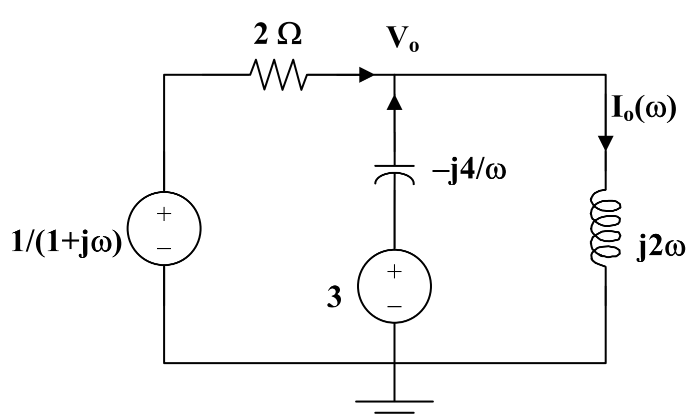
Chapter 18, Solution 46.

$$\frac{1}{4}F \longrightarrow \frac{1}{j\omega\frac{1}{4}} = \frac{-j4}{\omega}$$

$$2H \longrightarrow j\omega 2$$
$$3\delta(t) \longrightarrow 3$$

$$e^{-t}u(t) \longrightarrow \frac{1}{1 + j\omega}$$

The circuit in the frequency domain is shown below:



At node V_o, KCL gives

$$\frac{\frac{1}{1+j\omega} - V_o}{2} + \frac{3 - V_o}{\frac{-j4}{\omega}} = \frac{V_o}{j2\omega}$$

$$\frac{2}{1+j\omega} - 2V_o + j\omega 3 - j\omega V_o = -\frac{j2V_o}{\omega}$$

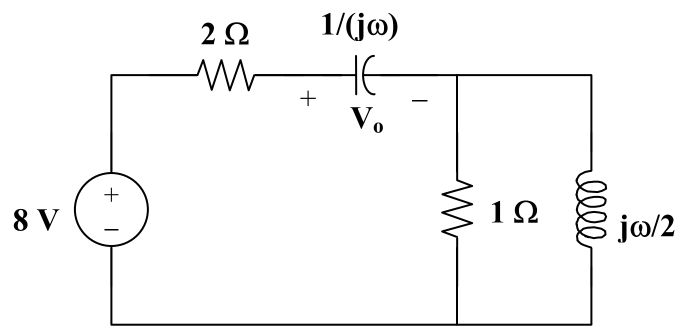
$$V_o = \frac{\frac{2}{1+j\omega} + j\omega 3}{2 + j\omega - \frac{j2}{\omega}}$$

$$I_o(\omega) = \frac{V_o}{j2\omega} = \frac{\frac{2 + j\omega 3 - 3\omega^2}{1+j\omega}}{j2\omega\left(2 + j\omega - \frac{j2}{\omega}\right)}$$

$$I_o(\omega) \;=\; \underline{\frac{\mathbf{2 + j\omega^2 - 3\omega^2}}{\mathbf{4 - 6\omega^2 + j(8\omega - 2\omega^3)}}}$$

Chapter 18, Solution 47.

Transferring the current source to a voltage source gives the circuit below:



Let $Z_{in} = 2 + 1 \| \frac{j\omega}{2} = 2 + \frac{\frac{j\omega}{2}}{1 + \frac{j\omega}{2}} = \frac{4 + j3\omega}{2 + j\omega}$

By voltage division,

$$V_o(\omega) = \frac{\frac{1}{j\omega}}{\frac{1}{j\omega} + Z_{in}} \cdot 8 = \frac{8}{1 + j\omega Z_{in}} = \frac{8}{1 + \frac{j\omega(4 + j3\omega)}{2 + j\omega}}$$

$$= \frac{8(2 + j\omega)}{2 + j\omega + j\omega 4 - 3\omega^2}$$

$$= \underline{\boldsymbol{\frac{8(2 + j\omega)}{2 + j\omega 5 - 3\omega^2}}}$$

Chapter 18, Solution 48.

$$0.2\text{F} \longrightarrow \frac{1}{j\omega C} = -\frac{j5}{\omega}$$

As an integrator,

$RC = 20 \text{ x } 10^3 \text{ x } 20 \text{ x } 10^{-6} = 0.4$

$$v_o = -\frac{1}{RC}\int_o^t v_i dt$$

$$V_o = -\frac{1}{RC}\left[\frac{V_i}{j\omega} + \pi V_i(0)\delta(\omega)\right]$$

$$= -\frac{1}{0.4}\left[\frac{2}{j\omega(2 + j\omega)} + \pi\delta(\omega)\right]$$

$$I_o = \frac{V_o}{20}\text{mA} = -0.125\left[\frac{2}{j\omega(2 + j\omega)} + \pi\delta(\omega)\right]$$

$$= -\frac{0.125}{j\omega} + \frac{0.125}{2 + j\omega} - 0.125\pi\delta(\omega)$$