Chapter 17, Solution 1.

(a) This is **periodic** with $\omega = \pi$ which leads to $T = 2\pi/\omega = \underline{\mathbf{2}}$.

(b) y(t) is **not periodic** although sin t and 4 cos 2πt are independently periodic.

(c) Since sin A cos B = 0.5[sin(A + B) + sin(A – B)],

$$g(t) = \sin 3t \cos 4t = 0.5[\sin 7t + \sin(-t)] = -0.5 \sin t + 0.5 \sin 7t$$

which is harmonic or **periodic** with the fundamental frequency

$$\omega = 1 \text{ or } T = 2\pi/\omega = \underline{\mathbf{2\pi}}.$$

(d) $h(t) = \cos^2 t = 0.5(1 + \cos 2t)$. Since the sum of a periodic function and a constant is also **periodic**, h(t) is periodic. $\omega = 2$ or $T = 2\pi/\omega = \underline{\pi}$.

(e) The frequency ratio $0.6|0.4 = 1.5$ makes z(t) **periodic**.

$$\omega = 0.2\pi \text{ or } T = 2\pi/\omega = \underline{\mathbf{10}}.$$

(f) p(t) = 10 is **not periodic**.

(g) g(t) is **not periodic**.

Chapter 17, Solution 2.

(a) The frequency ratio is 6/5 = 1.2. The highest common factor is 1.

$$\omega = 1 = 2\pi/T \text{ or } T = \underline{\mathbf{2\pi}}.$$

(b) $\omega = 2$ or $T = 2\pi/\omega = \underline{\pi}$.

(c) $f_3(t) = 4\sin^2 600\pi t = (4/2)(1 - \cos 1200\pi t)$

$$\omega = 1200\pi \text{ or } T = 2\pi/\omega = 2\pi/(1200\pi) = \underline{\mathbf{1/600}}.$$

(d) $f_4(t) = e^{j10t} = \cos 10t + j\sin 10t$. $\omega = 10$ or $T = 2\pi/\omega = \underline{\mathbf{0.2\pi}}$.

Chapter 17, Solution 3.

$T = 4,\ \omega_o = 2\pi/T = \pi/2$

$$g(t) = \begin{cases} 5, & 0 < t < 1 \\ 10, & 1 < t < 2 \\ 0, & 2 < t < 4 \end{cases}$$

$$a_o = (1/T)\int_0^T g(t)dt = 0.25[\int_0^1 5dt + \int_1^2 10dt] = \underline{\mathbf{3.75}}$$

$$a_n = (2/T)\int_0^T g(t)\cos(n\omega_o t)dt = (2/4)[\int_0^1 5\cos(\frac{n\pi}{2}t)dt + \int_1^2 10\cos(\frac{n\pi}{2}t)dt]$$

$$= 0.5[5\frac{2}{n\pi}\sin\frac{n\pi}{2}t\Big|_0^1 + 10\frac{2}{n\pi}\sin\frac{n\pi}{2}t\Big|_1^2] = (-1/(n\pi))5\sin(n\pi/2)$$

$$a_n = \begin{cases} \underline{\mathbf{(5/(n\pi))(-1)^{(n+1)/2},}} & \underline{\mathbf{n = odd}} \\ \underline{\mathbf{0,}} & \underline{\mathbf{n = even}} \end{cases}$$

$$b_n = (2/T)\int_0^T g(t)\sin(n\omega_o t)dt = (2/4)[\int_0^1 5\sin(\frac{n\pi}{2}t)dt + \int_1^2 10\sin(\frac{n\pi}{2}t)dt]$$

$$= 0.5[\frac{-2x5}{n\pi}\cos\frac{n\pi}{2}t\Big|_0^1 - \frac{2x10}{n\pi}\cos\frac{n\pi}{2}t\Big|_1^2] = \underline{\mathbf{(5/(n\pi))[3 - 2\cos n\pi + \cos(n\pi/2)]}}$$

Chapter 17, Solution 4.

$f(t) = 10 - 5t,\ 0 < t < 2,\ T = 2,\ \omega_o = 2\pi/T = \pi$

$$a_o = (1/T)\int_0^T f(t)dt = (1/2)\int_0^2 (10 - 5t)dt = 0.5[10t - (5t^2/2)]\Big|_0^2 = 5$$

$$a_n = (2/T)\int_0^T f(t)\cos(n\omega_o t)dt = (2/2)\int_0^2 (10 - 5t)\cos(n\pi t)dt$$

$$= \int_0^2 (10)\cos(n\pi t)dt - \int_0^2 (5t)\cos(n\pi t)dt$$

$$= \frac{-5}{n^2\pi^2}\cos n\pi t\Big|_0^2 + \frac{5t}{n\pi}\sin n\pi t\Big|_0^2 = [-5/(n^2\pi^2)](\cos 2n\pi - 1) = 0$$

$$b_n = (2/2)\int_0^2 (10 - 5t)\sin(n\pi t)dt$$

$$= \int_0^2 (10)\sin(n\pi t)dt - \int_0^2 (5t)\sin(n\pi t)dt$$

$$= \frac{-5}{n^2\pi^2}\sin n\pi t\Big|_0^2 + \frac{5t}{n\pi}\cos n\pi t\Big|_0^2 = 0 + [10/(n\pi)](\cos 2n\pi) = 10/(n\pi)$$

Hence $\quad f(t) = \mathbf{5 + \frac{10}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(n\pi t)}$

Chapter 17, Solution 5.

$T = 2\pi, \quad \omega = 2\pi / T = 1$

$$a_o = \frac{1}{T}\int_0^T z(t)dt = \frac{1}{2\pi}[1x\pi - 2x\pi] = -0.5$$

$$a_n = \frac{2}{T}\int_0^T z(t)\cos n\omega_o dt = \frac{1}{\pi}\int_0^{\pi} 1\cos ntdt - \frac{1}{\pi}\int_{\pi}^{2\pi} 2\cos ntdt = \frac{1}{n\pi}\sin ..nt\ \Big|_0^{\pi} - \frac{2}{n\pi}\sin nt\ \Big|_{\pi}^{2\pi} = 0$$

$$b_n = \frac{2}{T}\int_0^T z(t)\cos n\omega_o dt = \frac{1}{\pi}\int_0^{\pi} 1\sin ntdt - \frac{1}{\pi}\int_{\pi}^{2\pi} 2\sin ntdt = -\frac{1}{n\pi}\cos nt\ \Big|_0^{\pi} + \frac{2}{n\pi}\cos nt\ \Big|_{\pi}^{2\pi} = \begin{cases}\frac{6}{n\pi}, n = \text{odd}\\ 0, \quad n = \text{even}\end{cases}$$

Thus,

$$z(t) = -0.5 + \sum_{\substack{n=1\\ n=odd}}^{\infty}\frac{6}{n\pi}\sin nt$$

Chapter 17, Solution 6.

$$T = 2,\ \omega_o = \frac{2\pi}{2} = \pi$$

$$a_o = \frac{1}{2}\int_0^2 y(t)dt = \frac{1}{2}(4x1 + 2x1) = \frac{6}{2} = 3$$

Since this is an odd function, $a_n = 0.$

$$b_n = \frac{2}{2}\int_0^2 y(t)\sin(n\omega_o t)dt = \int_0^1 4\sin(n\pi t)dt + \int_1^2 2\sin(n\pi t)dt$$

$$= \frac{-4}{n\pi}\cos(n\pi t)\Big|_0^1 - \frac{2}{n\pi}\cos(n\pi t)\Big|_1^2 = \frac{-4}{n\pi}(\cos(n\pi) - 1) - \frac{2}{n\pi}(\cos(2n\pi) - \cos(n\pi))$$

$$= \frac{4}{n\pi}(1 - \cos(n\pi)) - \frac{2}{n\pi}(1 - \cos(n\pi)) = \frac{2}{n\pi}(1 - \cos(n\pi)) = \begin{cases} 0, & n = \text{even} \\ \frac{4}{n\pi}, & n = \text{odd} \end{cases}$$

$$y(t) = 3 + \frac{4}{\pi}\sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \frac{1}{n}\sin(n\pi t)$$

Chapter 17, Solution 7.

$$T = 12, \qquad \omega = 2\pi / T = \frac{\pi}{6}, \qquad a_0 = 0$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o dt = \frac{1}{6}[\int_{-2}^{4} 10\cos n\pi t / 6dt + \int_4^{10}(-10)\cos n\pi t / 6dt]$$

$$= \frac{10}{n\pi}\sin n\pi t / 6\ \Big|_{-2}^{4} - \frac{10}{n\pi}\sin n\pi t / 6\ \Big|_4^{10} = \frac{10}{n\pi}\left[2\sin 2n\pi/3 + \sin n\pi/3 - \sin 5n\pi/3\right]$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o dt = \frac{1}{6}[\int_{-2}^{4} 10\sin n\pi t / 6dt + \int_4^{10}(-10)\sin n\pi t / 6dt]$$

$$= -\frac{10}{n\pi}\cos n\pi t/6 \Big|_{-2}^{4} + \frac{10}{n\pi}\cos n\pi t/6 \Big|_{4}^{10} = \frac{10}{n\pi}\left[\cos 5n\pi/3 + \cos n\pi/3 - 2\sin 2n\pi/3\right]$$

$$\underline{f(t) = \sum_{n=1}^{\infty}\left(a_n \cos n\pi t/6 + b_n \sin n\pi t/6\right)}$$

where a_n and b_n are defined above.

Chapter 17, Solution 8.

$$f(t) = 2(1+t), \ -1 < t < 1, \quad T = 2, \quad \omega_o = 2\pi/T = \pi$$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2}\int_{-1}^{1} 2(t+1)dt = t^2 + t \Big|_{-1}^{1} = 2$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o dt = \frac{2}{2}\int_{-1}^{1} 2(t+1)\cos n\pi t dt = 2\left(\frac{1}{n^2\pi^2}\cos n\pi t + \frac{t}{n\pi}\sin n\pi t + \frac{1}{n\pi}\sin n\pi t\right)\Bigg|_{-1}^{1} = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o dt = \frac{2}{2}\int_{-1}^{1} 2(t+1)\sin n\pi t dt = 2\left(-\frac{1}{n^2\pi^2}\sin n\pi t - \frac{t}{n\pi}\cos n\pi t - \frac{1}{n\pi}\cos n\pi t\right)\Bigg|_{-1}^{1} = -\frac{4}{n\pi}\cos n\pi$$

$$\underline{f(t) = 2 - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\cos n\pi t}$$

Chapter 17, Solution 9.

f(t) is an even function, b_n=0.

$$T = 8, \quad \omega = 2\pi/T = \pi/4$$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{2}{8}\left[\int_0^2 10\cos \pi t/4 dt + 0\right] = \frac{10}{4}\left(\frac{4}{\pi}\right)\sin \pi t/4 \Big|_0^2 = \frac{10}{\pi} = 3.183$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_o dt = \frac{40}{8}[\int_0^2 10\cos \pi t/4\cos n\pi t/4 dt + 0] = 5\int_0^2 [\cos \pi t(n+1)/4 + \cos \pi t(n-1)/4]dt$$

For n = 1,

$$a_1 = 5\int_0^2 [\cos \pi t/2 + 1]dt = 5\left[\frac{2}{\pi}\sin \pi t/2 dt + t\right]_0^2 = 10$$

For n>1,

$$a_n = \frac{20}{\pi(n+1)}\sin\frac{\pi(n+1)t}{4} + \frac{20}{\pi(n-1)}\sin\frac{\pi(n-1)}{4}\Big|_0^2 = \frac{20}{\pi(n+1)}\sin\frac{\pi(n+1)}{2} + \frac{20}{\pi(n-1)}\sin\frac{\pi(n-1)}{2}$$

$$a_2 = \frac{10}{\pi}\sin\pi + \frac{20}{\pi}\sin\pi/2 = 6.3662, \qquad a_3 = \frac{20}{4\pi}\sin 2\pi + \frac{10}{\pi}\sin\pi = 0$$

Thus,

$$\underline{a_0 = 3.183, \quad a_1 = 10, \quad a_2 = 6.362, \quad a_3 = 0, \quad b_1 = 0 = b_2 = b_3}$$

Chapter 17, Solution 10.

$$T = 2, \qquad \omega_o = 2\pi/T = \pi$$

$$c_n = \frac{1}{T}\int_0^T h(t)e^{-jn\omega_o t}dt = \frac{1}{2}\left[\int_0^1 4e^{-jn\pi t}dt + \int_1^2 (-2)e^{-jn\pi t}dt\right] = \frac{1}{2}\left[\frac{4e^{-jn\pi t}}{-jn\pi}\Big|_0^1 - \frac{2e^{-jn\pi t}}{-jn\pi}\Big|_1^2\right]$$

$$c_n = \frac{j}{2n\pi}\left[4e^{-j\pi n} - 4 - 2e^{-j2n\pi} + 2e^{-jn\pi}\right] = \frac{j}{2n\pi}[6\cos n\pi - 6] = \begin{cases} -\frac{6j}{n\pi}, & n = \text{odd} \\ 0, & n = \text{even} \end{cases},$$

Thus,

$$\underline{f(t) = \sum_{\substack{n=-\infty \\ n=\text{odd}}}^{\infty} \left(\frac{-j6}{n\pi}\right)e^{jn\pi t}}$$

Chapter 17, Solution 11.

$T = 4, \quad \omega_o = 2\pi / T = \pi / 2$

$$c_n = \frac{1}{T}\int_0^T y(t)e^{-jn\omega_o t}dt = \frac{1}{4}\left[\int_{-1}^{0}(t+1)e^{-jn\pi t/2}dt + \int_0^1 (1)e^{-jn\pi t/2}dt\right]$$

$$c_n = \frac{1}{4}\left[\frac{e^{-jn\pi t/2}}{-n^2\pi^2/4}(-jn\pi t/2 - 1) - \frac{2}{jn\pi}e^{-jn\pi t/2}\Big|_{-1}^{0} - \frac{2}{jn\pi}e^{-jn\pi t/2}\Big|_{0}^{1}\right]$$

$$= \frac{1}{4}\left[\frac{4}{n^2\pi^2} - \frac{2}{jn\pi} + \frac{4}{n^2\pi^2}e^{jn\pi/2}(jn\pi/2 - 1) + \frac{2}{jn\pi}e^{jn\pi/2} - \frac{2}{jn\pi}e^{-jn\pi/2} + \frac{2}{jn\pi}\right]$$

But

$$e^{jn\pi/2} = \cos n\pi/2 + j\sin n\pi/2 = j\sin n\pi/2, \qquad e^{-jn\pi/2} = \cos n\pi/2 - j\sin n\pi/2 = -j\sin n\pi/2$$

$$c_n = \frac{1}{n^2\pi^2}\left[1 + j(jn\pi/2 - 1)\sin n\pi/2 + n\pi\sin n\pi/2\right]$$

$$\underline{y(t) = \sum_{n=-\infty}^{\infty}\frac{1}{n^2\pi^2}\left[1 + j(jn\pi/2 - 1)\sin n\pi/2 + n\pi\sin n\pi/2\right]e^{jn\pi t/2}}$$

Chapter 17, Solution 12.

A voltage source has a periodic waveform defined over its period as
$v(t) = t(2\pi - t)$ V, for all $0 < t < 2\pi$

Find the Fourier series for this voltage.

$v(t) = 2\pi t - t^2$, $0 < t < 2\pi$, $T = 2\pi$, $\omega_o = 2\pi/T = 1$

$a_o =$

$$(1/T)\int_0^T f(t)dt = \frac{1}{2\pi}\int_0^{2\pi}(2\pi t - t^2)dt = \frac{1}{2\pi}(\pi t^2 - t^3/3)\Big|_0^{2\pi} = \frac{4\pi^3}{2\pi}(1 - 2/3) = \frac{2\pi^2}{3}$$

Chapter 17, Solution 14.

Since cos(A + B) = cos A cos B – sin A sin B.

$$f(t) = \underline{2+\sum_{n=1}^{\infty}\left(\frac{10}{n^3+1}\cos(n\pi/4)\cos(2nt)-\frac{10}{n^3+1}\sin(n\pi/4)\sin(2nt)\right)}$$

Chapter 17, Solution 15.

(a) $D\cos\omega t + E\sin\omega t = A\cos(\omega t - \theta)$

where $A = \sqrt{D^2+E^2}$, $\theta = \tan^{-1}(E/D)$

$$A = \sqrt{\frac{16}{(n^2+1)^2}+\frac{1}{n^6}},\quad \theta = \tan^{-1}((n^2+1)/(4n^3))$$

$$f(t) = \underline{10 + \sum_{n=1}^{\infty}\sqrt{\frac{16}{(n^2+1)^2}+\frac{1}{n^6}}\cos\left(10nt - \tan^{-1}\frac{n^2+1}{4n^3}\right)}$$

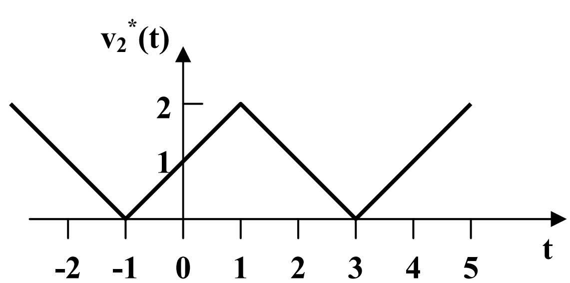
(b) $D\cos\omega t + E\sin\omega t = A\sin(\omega t + \theta)$

where $A = \sqrt{D^2+E^2}$, $\theta = \tan^{-1}(D/E)$

$$f(t) = \underline{10 + \sum_{n=1}^{\infty}\sqrt{\frac{16}{(n^2+1)^2}+\frac{1}{n^6}}\sin\left(10nt + \tan^{-1}\frac{4n^3}{n^2+1}\right)}$$

Chapter 17, Solution 16.

If $v_2(t)$ is shifted by 1 along the vertical axis, we obtain $v_2^*(t)$ shown below, i.e. $v_2^*(t) = v_2(t) + 1$.



Comparing $v_2^*(t)$ with $v_1(t)$ shows that

$$v_2^*(t) = 2v_1((t + t_o)/2)$$

$$\text{where } (t + t_o)/2 = 0 \text{ at } t = -1 \text{ or } t_o = 1$$

Hence $$v_2^*(t) = 2v_1((t + 1)/2)$$

But $$v_2^*(t) = v_2(t) + 1$$

$$v_2(t) + 1 = 2v_1((t+1)/2)$$

$$v_2(t) = -1 + 2v_1((t+1)/2)$$

$$= -1 + 1 - \frac{8}{\pi^2}\left[\cos \pi\left(\frac{t+1}{2}\right) + \frac{1}{9}\cos 3\pi\left(\frac{t+1}{2}\right) + \frac{1}{25}\cos 5\pi\left(\frac{t+1}{2}\right) + \cdots\right]$$

$$v_2(t) = -\frac{8}{\pi^2}\left[\cos\left(\frac{\pi t}{2} + \frac{\pi}{2}\right) + \frac{1}{9}\cos\left(\frac{3\pi t}{2} + \frac{3\pi}{2}\right) + \frac{1}{25}\cos\left(\frac{5\pi t}{2} + \frac{5\pi}{2}\right) + \cdots\right]$$

$$v_2(t) = \underline{-\frac{8}{\pi^2}\left[\sin\left(\frac{\pi t}{2}\right) + \frac{1}{9}\sin\left(\frac{3\pi t}{2}\right) + \frac{1}{25}\sin\left(\frac{5\pi t}{2}\right) + \cdots\right]}$$

Chapter 17, Solution 17.

We replace t by –t in each case and see if the function remains unchanged.

(a) $1 - t$, **neither odd nor even.**

(b) $t^2 - 1$, **even**

(c) $\cos n\pi(-t) \sin n\pi(-t) = -\cos n\pi t \sin n\pi t$, **odd**

(d) $\sin^2 n(-t) = (-\sin \pi t)^2 = \sin^2 \pi t$, **even**

(e) e^t, **neither odd nor even**.

Chapter 17, Solution 18.

(a) $T = 2$ leads to $\omega_o = 2\pi/T = \underline{\pi}$

$f_1(-t) = -f_1(t)$, showing that $f_1(t)$ is **<u>odd and half-wave symmetric</u>**.

(b) $T = 3$ leads to $\omega_o = 2\pi/3$

$f_2(t) = f_2(-t)$, showing that $f_2(t)$ is **<u>even</u>**.

(c) $T = 4$ leads to $\omega_o = \pi/2$

$f_3(t)$ is **<u>even and half-wave symmetric</u>**.

Chapter 17, Solution 19.

This is a half-wave even symmetric function.

$$a_o = 0 = b_n, \quad \omega_o = 2\pi/T \quad \pi/2$$

$$a_n = \frac{4}{T}\int_0^{T/2}\left[1 - \frac{4t}{T}\right]\cos(n\omega_o t)dt$$

$$= [4/(n\pi)^2](1 - \cos n\pi) \quad = 8/(n^2\pi^2), \quad n = \text{odd}$$
$$= 0, \quad n = \text{even}$$

$$\underline{\mathbf{f(t) = \frac{8}{\pi^2}\sum_{n=odd}^{\infty}\frac{1}{n^2}\cos\left(\frac{n\pi t}{2}\right)}}$$

Chapter 17, Solution 20.

This is an even function.

$$b_n = 0, \ T = 6, \ \omega = 2\pi/6 = \pi/3$$

$$a_o = \frac{2}{T}\int_0^{T/2} f(t)dt = \frac{2}{6}\left[\int_1^2(4t - 4)dt\int_2^3 4\,dt\right]$$