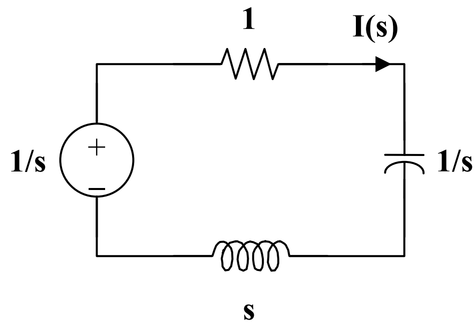
Chapter 16, Solution 1.

Consider the s-domain form of the circuit which is shown below.

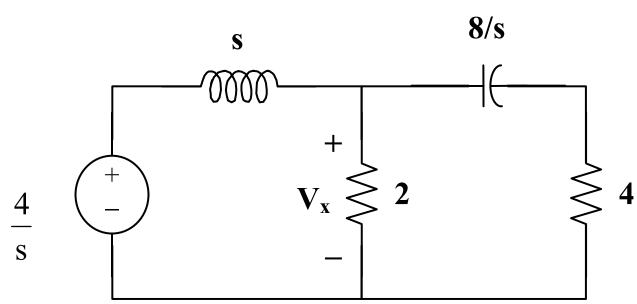


$$I(s) = \frac{1/s}{1+s+1/s} = \frac{1}{s^2+s+1} = \frac{1}{(s+1/2)^2+(\sqrt{3}/2)^2}$$

$$i(t) = \frac{2}{\sqrt{3}} e^{-t/2} \sin\left(\frac{\sqrt{3}}{2}t\right)$$

$$i(t) = \underline{\mathbf{1.155 e^{-0.5t} \sin(0.866t)\ A}}$$

Chapter 16, Solution 2.



$$\frac{V_x - \frac{4}{s}}{s} + \frac{V_x - 0}{2} + \frac{V_x - 0}{4 + \frac{8}{s}} = 0$$

$$V_x(4s + 8) - \frac{(16s + 32)}{s} + (2s^2 + 4s)V_x + s^2 V_x = 0$$

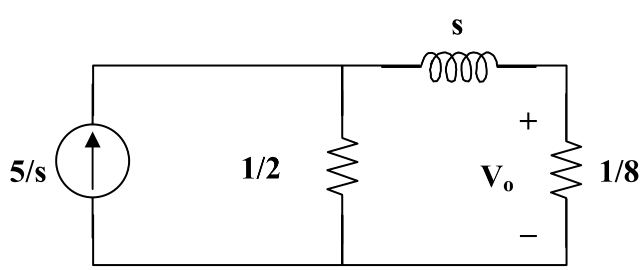
$$V_x(3s^2 + 8s + 8) = \frac{16s + 32}{s}$$

$$V_x = -16\frac{s+2}{s(3s^2 + 8s + 8)} = -16\left(\frac{0.25}{s} + \frac{-0.125}{s + \frac{4}{3} + j\frac{\sqrt{8}}{3}} + \frac{-0.125}{s + \frac{4}{3} - j\frac{\sqrt{8}}{3}}\right)$$

$$v_x = \underline{(-4 + 2e^{-(1.3333 + j0.9428)t} + 2e^{-(1.3333 - j0.9428)t})u(t)\,V}$$

$$v_x = \underline{4u(t) - e^{-4t/3}\cos\left(\frac{2\sqrt{2}}{3}t\right) - \frac{6}{\sqrt{2}}e^{-4t/3}\sin\left(\frac{2\sqrt{2}}{3}t\right)\ V}$$

Chapter 16, Solution 3.



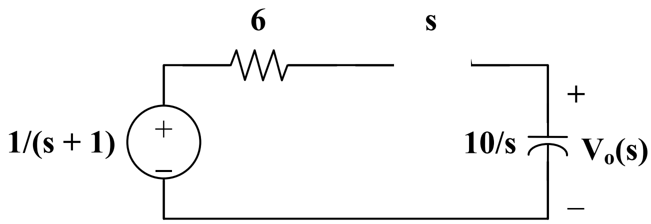
Current division leads to:

$$V_o = \frac{1}{8}\frac{5}{s}\left(\frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{8} + s}\right) = \frac{5}{10 + 16s} = \frac{5}{16(s + 0.625)}$$

$$v_o(t) = \underline{0.3125\left(1 - e^{-0.625t}\right)u(t)\ V}$$

Chapter 16, Solution 4.

The s-domain form of the circuit is shown below.



Using voltage division,

$$V_o(s) = \frac{10/s}{s+6+10/s}\left(\frac{1}{s+1}\right) = \frac{10}{s^2+6s+10}\left(\frac{1}{s+1}\right)$$

$$V_o(s) = \frac{10}{(s+1)(s^2+6s+10)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+6s+10}$$

$$10 = A(s^2+6s+10) + B(s^2+s) + C(s+1)$$

Equating coefficients :

s^2 : $0 = A + B \longrightarrow B = \text{-}A$

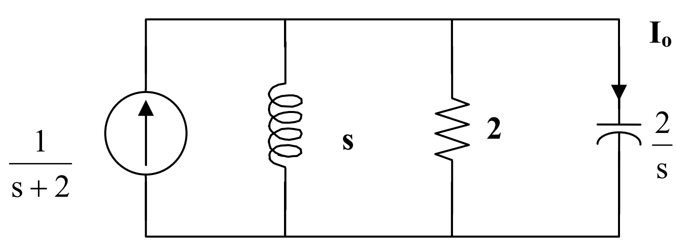
s^1 : $0 = 6A + B + C = 5A + C \longrightarrow C = \text{-}5A$

s^0 : $10 = 10A + C = 5A \longrightarrow A = 2, B = \text{-}2, C = \text{-}10$

$$V_o(s) = \frac{2}{s+1} - \frac{2s+10}{s^2+6s+10} = \frac{2}{s+1} - \frac{2(s+3)}{(s+3)^2+1^2} - \frac{4}{(s+3)^2+1^2}$$

$$v_o(t) = \underline{\mathbf{2e^{-t} - 2e^{-3t}\cos(t) - 4e^{-3t}\sin(t)\ V}}$$

Chapter 16, Solution 5.



$$V = \frac{1}{s+2}\left(\frac{1}{\frac{1}{s}+\frac{1}{2}+\frac{s}{2}}\right) = \frac{1}{s+2}\left(\frac{2s}{s^2+s+2}\right) = \frac{2s}{(s+2)(s+0.5+j1.3229)(s+0.5-j1.3229)}$$

$$I_o = \frac{Vs}{2} = \frac{s^2}{(s+2)(s+0.5+j1.3229)(s+0.5-j1.3229)}$$

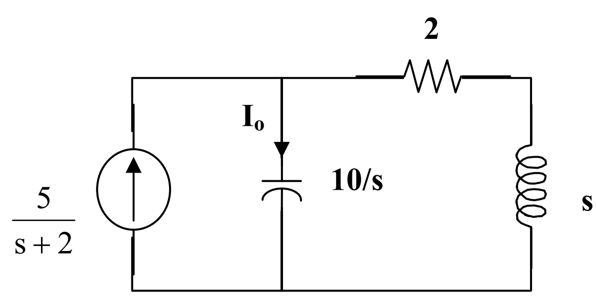
$$= \frac{1}{s+2} + \frac{\frac{(-0.5-j1.3229)^2}{(1.5-j1.3229)(-j2.646)}}{s+0.5+j1.3229} + \frac{\frac{(-0.5+j1.3229)^2}{(1.5+j1.3229)(+j2.646)}}{s+0.5-j1.3229}$$

$$\underline{i_o(t) = \left(e^{-2t} + 0.3779e^{-90°}e^{-t/2}e^{-j1.3229t} + 0.3779e^{90°}e^{-t/2}e^{j1.3229t}\right)u(t)\,A}$$

or

$$\underline{= \left(e^{-2t} - 0.7559\sin 1.3229t\right)u(t)\,A}$$

Chapter 16, Solution 6.



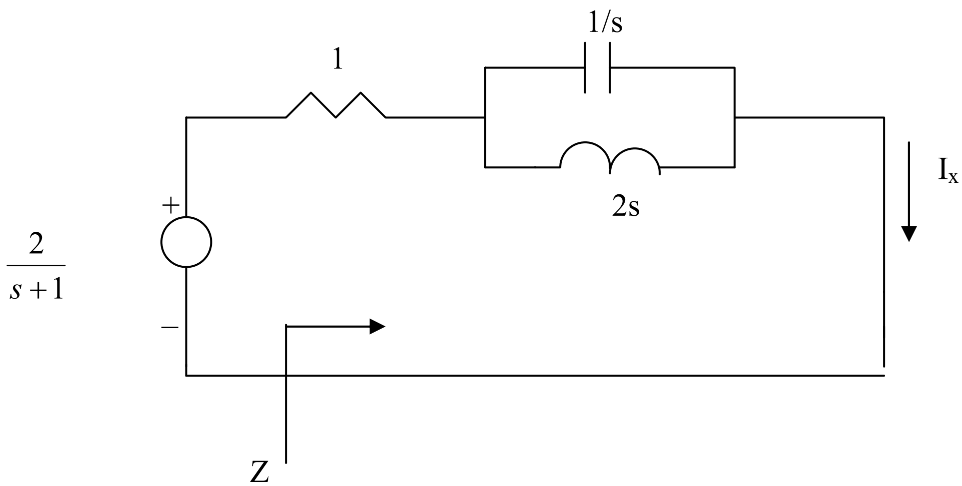
Use current division.

$$I_o = \frac{s+2}{s+2+\frac{10}{s}}\frac{5}{s+2} = \frac{5s}{s^2+2s+10} = \frac{5(s+1)}{(s+1)^2+3^2} - \frac{5}{(s+1)^2+3^2}$$

$$\underline{i_o(t) = 5e^{-t}\cos 3t - \frac{5}{3}e^{-t}\sin 3t}$$

Chapter 16, Solution 7.

The s-domain version of the circuit is shown below.



$$Z = 1 + \frac{1}{s} // 2s = 1 + \frac{\frac{1}{s}(2s)}{\frac{1}{s} + 2s} = 1 + \frac{2s}{1+2s^2} = \frac{2s^2 + 2s + 1}{1 + 2s^2}$$

$$I_x = \frac{V}{Z} = \frac{2}{s+1} \text{ x } \frac{1+2s^2}{2s^2+2s+1} = \frac{2s^2+1}{(s+1)(s^2+s+0.5)} = \frac{A}{(s+1)} + \frac{Bs+C}{(s^2+s+0.5)}$$

$$2s^2 + 1 = A(s^2 + s + 0.5) + B(s^2 + s) + C(s+1)$$

s^2 : $\quad 2 = A + B$

s : $\quad 0 = A + B + C = 2 + C \longrightarrow C = -2$

constant : $\quad 1 = 0.5A + C$ or $0.5A = 3 \longrightarrow A = 6, B = -4$

$$I_x = \frac{6}{s+1} - \frac{4s+2}{(s+0.5)^2 + 0.75} = \frac{6}{s+1} - \frac{4(s+0.5)}{(s+0.5)^2 + 0.866^2}$$

$$i_x(t) = \underline{\left[6 - 4e^{-0.5t}\cos 0.866t\right]u(t)\,A}$$

Chapter 16, Solution 8.

(a) $Z = \frac{1}{s} + 1//(1+2s) = \frac{1}{s} + \frac{(1+2s)}{2+2s} = \underline{\frac{s^2 + 1.5s + 1}{s(s+1)}}$

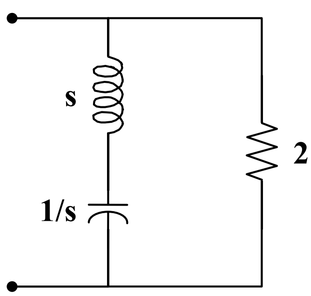
(b) $\frac{1}{Z} = \frac{1}{2} + \frac{1}{s} + \frac{1}{1+\frac{1}{s}} = \frac{3s^2 + 3s + 2}{2s(s+1)}$

$Z = \underline{\frac{2s(s+1)}{3s^2 + 3s + 2}}$

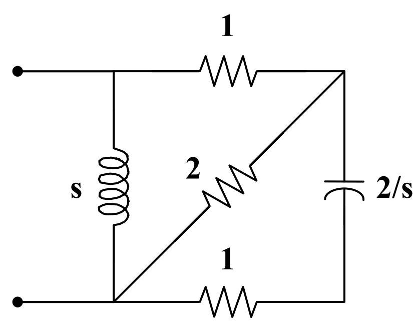
Chapter 16, Solution 9.

(a) The s-domain form of the circuit is shown in Fig. (a).

$$Z_{in} = 2 \,||\, (s + 1/s) = \frac{2(s+1/s)}{2+s+1/s} = \underline{\mathbf{\frac{2(s^2+1)}{s^2+2s+1}}}$$



(a)



(b)

(b) The s-domain equivalent circuit is shown in Fig. (b).

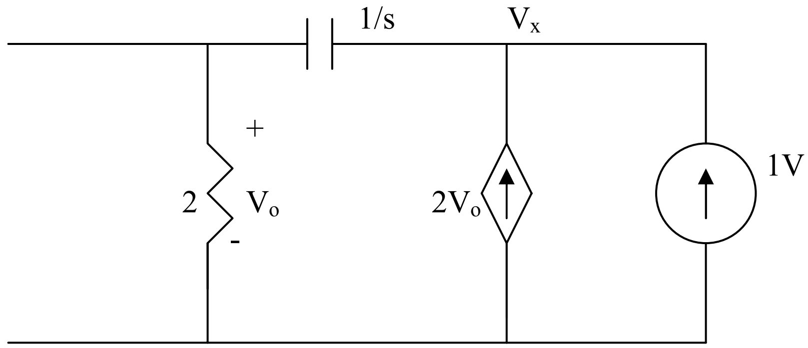
$$2 \,||\, (1 + 2/s) = \frac{2(1+2/s)}{3+2/s} = \frac{2(s+2)}{3s+2}$$

$$1 + 2 \,||\, (1+2/s) = \frac{5s+6}{3s+2}$$

$$Z_{in} = s \,||\left(\frac{5s+6}{3s+2}\right) = \frac{s \cdot \left(\frac{5s+6}{3s+2}\right)}{s + \left(\frac{5s+6}{3s+2}\right)} = \underline{\mathbf{\frac{s(5s+6)}{3s^2+7s+6}}}$$

Chapter 16, Solution 10.

To find Z_{Th}, consider the circuit below.



Applying KCL gives

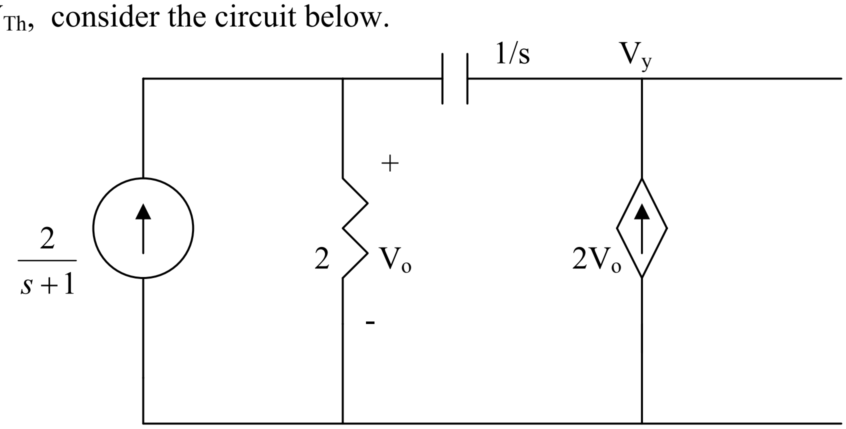
$$1 + 2V_o = \frac{V_x}{2 + 1/s}$$

But $V_o = \dfrac{2}{2 + 1/s} V_x$. Hence

$$1 + \frac{4V_x}{2 + 1/s} = \frac{V_x}{2 + 1/s} \quad \longrightarrow \quad V_x = -\frac{(2s+1)}{3s}$$

$$Z_{Th} = \frac{V_x}{1} = -\frac{(2s+1)}{3s}$$

To find V_{Th}, consider the circuit below.



Applying KCL gives

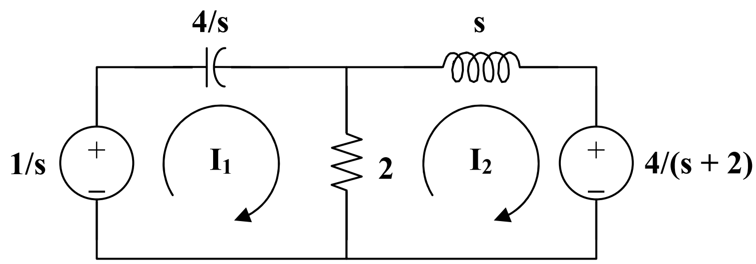
$$\frac{2}{s+1} + 2V_o = \frac{V_o}{2} \quad \longrightarrow \quad V_o = -\frac{4}{3(s+1)}$$

But $-V_y + 2V_o \bullet \frac{1}{s} + V_o = 0$

$$V_{Th} = V_y = V_o(1+\frac{2}{s}) = -\frac{4}{3(s+1)}\left(\frac{s+2}{s}\right) = \underline{\frac{-4(s+2)}{3s(s+1)}}$$

Chapter 16, Solution 11.

The s-domain form of the circuit is shown below.



Write the mesh equations.

$$\frac{1}{s} = \left(2+\frac{4}{s}\right) I_1 - 2I_2 \qquad (1)$$

$$\frac{-4}{s+2} = -2I_1 + (s+2)\,I_2 \qquad (2)$$

Put equations (1) and (2) into matrix form.

$$\begin{bmatrix} 1/s \\ -4/(s+2) \end{bmatrix} = \begin{bmatrix} 2+4/s & -2 \\ -2 & s+2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = \frac{2}{s}(s^2+2s+4), \quad \Delta_1 = \frac{s^2-4s+4}{s(s+2)}, \qquad \Delta_2 = \frac{-6}{s}$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{1/2\cdot(s^2-4s+4)}{(s+2)(s^2+2s+4)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+2s+4}$$

$$1/2\cdot(s^2-4s+4) = A(s^2+2s+4) + B(s^2+2s) + C(s+2)$$

Equating coefficients :

s^2: $\quad 1/2 = A + B$

s^1: $\quad -2 = 2A + 2B + C$

$s^0: \qquad 2 = 4A + 2C$

Solving these equations leads to $\quad A = 2, \quad B = -3/2, \quad C = -3$

$$I_1 = \frac{2}{s+2} + \frac{-3/2\,s - 3}{(s+1)^2 + (\sqrt{3})^2}$$

$$I_1 = \frac{2}{s+2} + \frac{-3}{2} \cdot \frac{(s+1)}{(s+1)^2 + (\sqrt{3})^2} + \frac{-3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{(s+1)^2 + (\sqrt{3})^2}$$

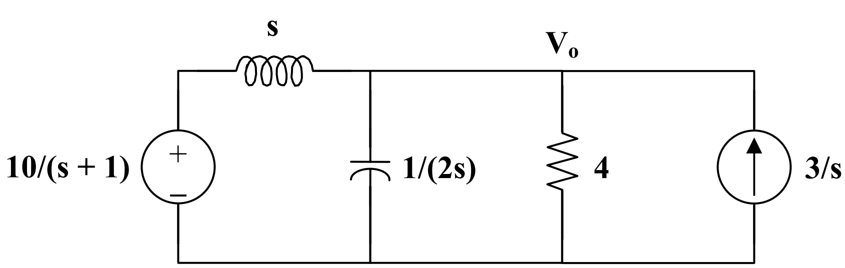
$$i_1(t) = \underline{\left[\mathbf{2\,e^{-2t} - 1.5\,e^{-t}\cos(1.732t) - 0.866\sin(1.732t)}\right]\mathbf{u(t)\ A}}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-6}{s} \cdot \frac{s}{2(s^2 + 2s + 4)} = \frac{-3}{(s+1)^2 + (\sqrt{3})^2}$$

$$i_2(t) = \frac{-3}{\sqrt{3}} e^{-t} \sin(\sqrt{3}t) = \underline{\mathbf{-1.732\,e^{-t}\sin(1.732t)\,u(t)\ A}}$$

Chapter 16, Solution 12.

We apply nodal analysis to the s-domain form of the circuit below.



$$\frac{\frac{10}{s+1} - V_o}{s} + \frac{3}{s} = \frac{V_o}{4} + 2sV_o$$

$$(1 + 0.25s + s^2)\,V_o = \frac{10}{s+1} + 15 = \frac{10 + 15s + 15}{s+1}$$

$$V_o = \frac{15s + 25}{(s+1)(s^2 + 0.25s + 1)} = \frac{A}{s+1} + \frac{Bs + C}{s^2 + 0.25s + 1}$$

$$A = (s+1)V_o\Big|_{s=-1} = \frac{40}{7}$$

$$15s + 25 = A(s^2 + 0.25s + 1) + B(s^2 + s) + C(s+1)$$

Equating coefficients :

s^2: $\quad 0 = A + B \longrightarrow B = -A$

s^1: $\quad 15 = 0.25A + B + C = -0.75A + C$

s^0: $\quad 25 = A + C$

$A = 40/7, \qquad B = -40/7, \qquad C = 135/7$

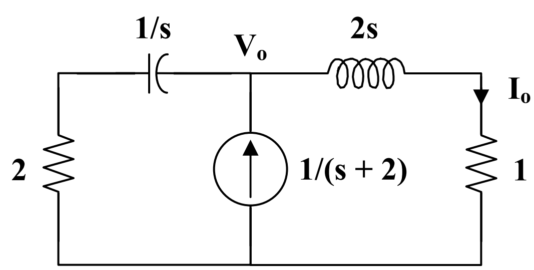
$$V_o = \frac{\frac{40}{7}}{s+1} + \frac{\frac{-40}{7}s + \frac{135}{7}}{\left(s+\frac{1}{2}\right)^2 + \frac{3}{4}} = \frac{40}{7}\frac{1}{s+1} - \frac{40}{7}\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2 + \frac{3}{4}} + \left(\frac{155}{7}\cdot\frac{2}{\sqrt{3}}\right)\frac{\frac{\sqrt{3}}{2}}{\left(s+\frac{1}{2}\right)^2 + \frac{3}{4}}$$

$$v_o(t) = \frac{40}{7}e^{-t} - \frac{40}{7}e^{-t/2}\cos\left(\frac{\sqrt{3}}{2}t\right) + \frac{(155)(2)}{(7)(\sqrt{3})}e^{-t/2}\sin\left(\frac{\sqrt{3}}{2}t\right)$$

$$v_o(t) = \underline{\mathbf{5.714\,e^{-t} - 5.714\,e^{-t/2}\cos(0.866t) + 25.57\,e^{-t/2}\sin(0.866t)\ V}}$$

Chapter 16, Solution 13.

Consider the following circuit.



Applying KCL at node o,

$$\frac{1}{s+2} = \frac{V_o}{2s+1} + \frac{V_o}{2+1/s} = \frac{s+1}{2s+1}V_o$$

$$\frac{V_o - 3V_x}{s/4} + \frac{V_o - 0}{5/s} + \frac{V_o - \frac{5}{s+2}}{10} = 0$$

$$40V_o - 120V_x + 2s^2V_o + sV_o - \frac{5s}{s+2} = 0 = (2s^2 + s + 40)V_o - 120V_x - \frac{5s}{s+2}$$

But, $V_x = V_o - \frac{5}{s+2} \rightarrow V_o = V_x + \frac{5}{s+2}$

We can now solve for V_x.

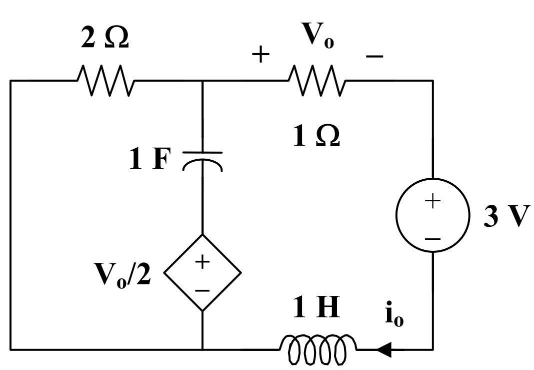
$$(2s^2 + s + 40)\left(V_x + \frac{5}{s+2}\right) - 120V_x - \frac{5s}{s+2} = 0$$

$$2(s^2 + 0.5s - 40)V_x = -10\frac{(s^2 + 20)}{s+2}$$

$$\underline{V_x = -5\frac{(s^2 + 20)}{(s+2)(s^2 + 0.5s - 40)}}$$

Chapter 16, Solution 16.

We first need to find the initial conditions. For $t < 0$, the circuit is shown in Fig. (a). To dc, the capacitor acts like an open circuit and the inductor acts like a short circuit.



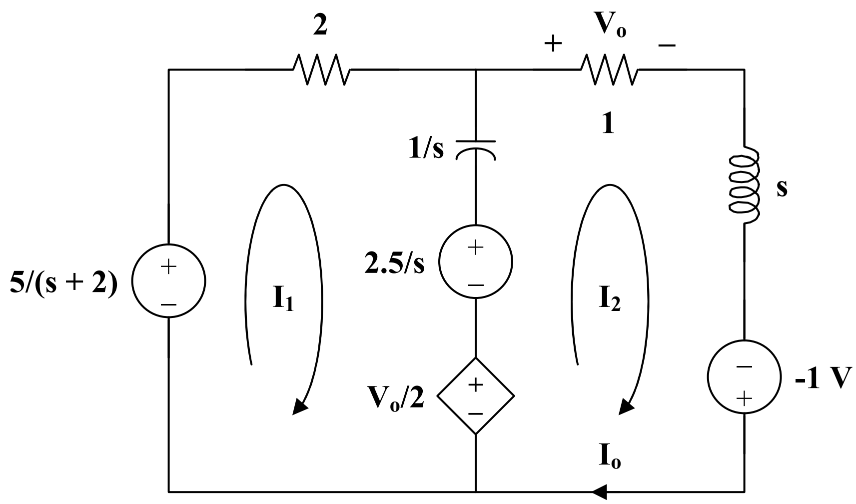
(a)

Hence,

$$i_L(0) = i_o = \frac{-3}{3} = -1\text{ A}, \qquad v_o = -1\text{ V}$$

$$v_c(0) = -(2)(-1) - \left(\frac{-1}{2}\right) = 2.5\text{ V}$$

We now incorporate the initial conditions for $t > 0$ as shown in Fig. (b).



(b)

For mesh 1,

$$\frac{-5}{s+2} + \left(2 + \frac{1}{s}\right)I_1 - \frac{1}{s}I_2 + \frac{2.5}{s} + \frac{V_o}{2} = 0$$

But, $\quad V_o = I_o = I_2$

$$\left(2 + \frac{1}{s}\right)I_1 + \left(\frac{1}{2} - \frac{1}{s}\right)I_2 = \frac{5}{s+2} - \frac{2.5}{s} \qquad (1)$$

For mesh 2,

$$\left(1 + s + \frac{1}{s}\right)I_2 - \frac{1}{s}I_1 + 1 - \frac{V_o}{2} - \frac{2.5}{s} = 0$$

$$-\frac{1}{s}I_1 + \left(\frac{1}{2} + s + \frac{1}{s}\right)I_2 = \frac{2.5}{s} - 1 \qquad (2)$$

Put (1) and (2) in matrix form.

$$\begin{bmatrix} 2+\frac{1}{s} & \frac{1}{2}-\frac{1}{s} \\ \frac{-1}{s} & \frac{1}{2}+s+\frac{1}{s} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{5}{s+2}-\frac{2.5}{s} \\ \frac{2.5}{s}-1 \end{bmatrix}$$

$$\Delta = 2s + 2 + \frac{3}{s}, \qquad \Delta_2 = -2 + \frac{4}{s} + \frac{5}{s(s+2)}$$

$$I_o = I_2 = \frac{\Delta_2}{\Delta} = \frac{-2s^2 + 13}{(s+2)(2s^2+2s+3)} = \frac{A}{s+2} + \frac{Bs+C}{2s^2+2s+3}$$

$$-2s^2 + 13 = A(2s^2+2s+3) + B(s^2+2s) + C(s+2)$$

Equating coefficients :

s^2: $\quad -2 = 2A + B$

s^1: $\quad 0 = 2A + 2B + C$

s^0: $\quad 13 = 3A + 2C$

Solving these equations leads to

$A = 0.7143, \quad B = -3.429, \quad C = 5.429$

$$I_o = \frac{0.7143}{s+2} - \frac{3.429s - 5.429}{2s^2+2s+3} = \frac{0.7143}{s+2} - \frac{1.7145s - 2.714}{s^2+s+1.5}$$

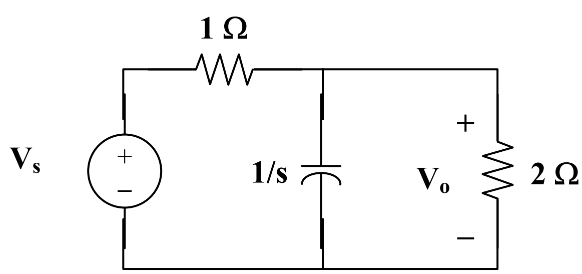
$$I_o = \frac{0.7143}{s+2} - \frac{1.7145(s+0.5)}{(s+0.5)^2+1.25} + \frac{(3.194)(\sqrt{1.25})}{(s+0.5)^2+1.25}$$

$$i_o(t) = \underline{\left[\mathbf{0.7143e^{-2t} - 1.7145e^{-0.5t}\cos(1.25t) + 3.194e^{-0.5t}\sin(1.25t)}\right]\mathbf{u(t)\ A}}$$

$$i_o(t) = i_2(t) = \underline{\mathbf{(2 - e^{-t})u(t)\ A}}$$

Chapter 16, Solution 18.

$v_s(t) = 3u(t) - 3u(t-1)$ or $V_s = \frac{3}{s} - \frac{e^{-s}}{s} = \frac{3}{s}(1 - e^{-s})$



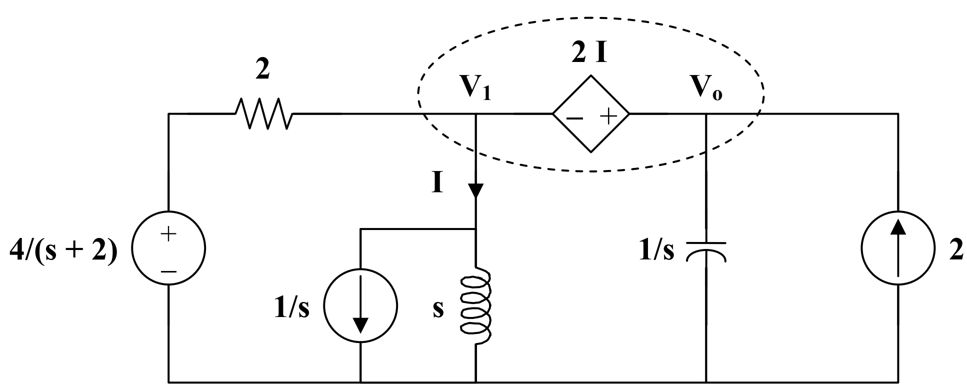
$$\frac{V_o - V_s}{1} + sV_o + \frac{V_o}{2} = 0 \rightarrow (s + 1.5)V_o = V_s$$

$$V_o = \frac{3}{s(s+1.5)}(1 - e^{-s}) = \left(\frac{2}{s} - \frac{2}{s+1.5}\right)(1 - e^{-s})$$

$$v_o(t) = \underline{[(2 - 2e^{-1.5t})u(t) - (2 - 2e^{-1.5(t-1)})u(t-1)]\,V}$$

Chapter 16, Solution 19.

We incorporate the initial conditions in the s-domain circuit as shown below.



At the supernode,

$$\frac{4/(s+2)-V_1}{2}+2=\frac{V_1}{s}+\frac{1}{s}+sV_o$$

$$\frac{2}{s+2}+2=\left(\frac{1}{2}+\frac{1}{s}\right)V_1+\frac{1}{s}+s\,V_o \qquad (1)$$

But $\quad V_o = V_1 + 2I \quad$ and $\qquad I=\dfrac{V_1+1}{s}$

$$V_o = V_1+\frac{2(V_1+1)}{s} \longrightarrow V_1=\frac{V_o-2/s}{(s+2)/s}=\frac{s\,V_o-2}{s+2} \qquad (2)$$

Substituting (2) into (1)

$$\frac{2}{s+2}+2-\frac{1}{s}=\left(\frac{2s+1}{s}\right)\left[\left(\frac{s}{s+2}\right)V_o-\frac{2}{s+2}\right]+s\,V_o$$

$$\frac{2}{s+2}+2-\frac{1}{s}+\frac{2(2s+1)}{s(s+2)}=\left[\left(\frac{2s+1}{s+2}\right)+s\right]V_o$$

$$\frac{2s^2+9s}{s(s+2)}=\frac{2s+9}{s+2}=\frac{s^2+4s+1}{s+2}V_o$$

$$V_o=\frac{2s+9}{s^2+4s+1}=\frac{A}{s+0.2679}+\frac{B}{s+3.732}$$

$A = 2.443, \qquad B = -0.4434$

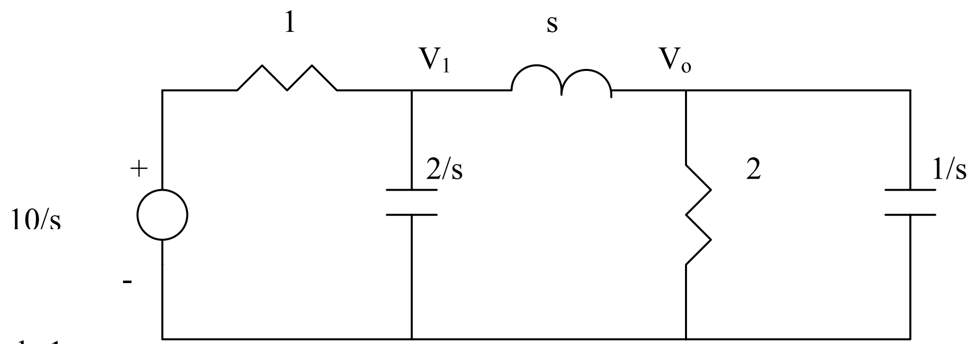
$$V_o=\frac{2.443}{s+0.2679}-\frac{0.4434}{s+3.732}$$

Therefore,

$v_o(t) =$ $\underline{\mathbf{(2.443\,e^{-0.2679t} - 0.4434\,e^{-3.732t})\,u(t)\ V}}$

Chapter 16, Solution 21.

The s-domain version of the circuit is shown below.



At node 1,

$$\frac{\frac{10}{s} - V_1}{1} = \frac{V_1 - V_o}{s} + \frac{s}{2}V_o \quad \longrightarrow \quad 10 = (s+1)V_1 + (\frac{s^2}{2} - 1)V_o \qquad (1)$$

At node 2,

$$\frac{V_1 - V_o}{s} = \frac{V_o}{2} + sV_o \quad \longrightarrow \quad V_1 = V_o(\frac{s}{2} + s^2 + 1) \qquad (2)$$

Substituting (2) into (1) gives

$$10 = (s+1)(s^2 + s/2 + 1)V_o + (\frac{s^2}{2} - 1)V_o = s(s^2 + 2s + 1.5)V_o$$

$$V_o = \frac{10}{s(s^2 + 2s + 1.5)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 2s + 1.5}$$

$$10 = A(s^2 + 2s + 1.5) + Bs^2 + Cs$$

$$s^2: \qquad 0 = A + B$$

$$s: \qquad 0 = 2A + C$$

$$\text{constant}: \qquad 10 = 1.5A \quad \longrightarrow \quad A = 20/3,\ \text{B} = \text{-20/3},\ \text{C} = \text{-40/3}$$

$$V_o = \frac{20}{3}\left[\frac{1}{s} - \frac{s+2}{s^2 + 2s + 1.5}\right] = \frac{20}{3}\left[\frac{1}{s} - \frac{s+1}{(s+1)^2 + 0.7071^2} - 1.414\frac{0.7071}{(s+1)^2 + 0.7071^2}\right]$$

Taking the inverse Laplace tranform finally yields

$$\underline{v_o(t) = \frac{20}{3}\left[1 - e^{-t}\cos 0.7071t - 1.414e^{-t}\sin 0.7071t\right]u(t)\,V}$$

$$I_o = \frac{10s+5}{(s+1)(s+1.5)} = \frac{A}{s+1} + \frac{B}{s+1.5}$$

$$A = -10, \qquad B = 20$$

$$I_o(s) = \frac{-10}{s+1} + \frac{20}{s+1.5}$$

$$i_o(t) = \underline{\mathbf{10\left[2e^{-1.5t} - e^{-t}\right]u(t)\ A}}$$

Chapter 16, Solution 30.

$$Y(s) = H(s)X(s), \qquad X(s) = \frac{4}{s+1/3} = \frac{12}{3s+1}$$

$$Y(s) = \frac{12s^2}{(3s+1)^2} = \frac{4}{3} - \frac{8s+4/3}{(3s+1)^2}$$

$$Y(s) = \frac{4}{3} - \frac{8}{9}\cdot\frac{s}{(s+1/3)^2} - \frac{4}{27}\cdot\frac{1}{(s+1/3)^2}$$

Let $G(s) = \frac{-8}{9}\cdot\frac{s}{(s+1/3)^2}$

Using the time differentiation property,

$$g(t) = \frac{-8}{9}\cdot\frac{d}{dt}(t\,e^{-t/3}) = \frac{-8}{9}\left(\frac{-1}{3}t\,e^{-t/3} + e^{-t/3}\right)$$

$$g(t) = \frac{8}{27}t\,e^{-t/3} - \frac{8}{9}e^{-t/3}$$

Hence,

$$y(t) = \frac{4}{3}u(t) + \frac{8}{27}t\,e^{-t/3} - \frac{8}{9}e^{-t/3} - \frac{4}{27}t\,e^{-t/3}$$

$$y(t) = \underline{\mathbf{\frac{4}{3}u(t) - \frac{8}{9}e^{-t/3} + \frac{4}{27}t\,e^{-t/3}}}$$

Chapter 16, Solution 31.

$$x(t) = u(t) \longrightarrow X(s) = \frac{1}{s}$$

$$y(t) = 10\cos(2t) \longrightarrow Y(s) = \frac{10s}{s^2 + 4}$$

$$H(s) = \frac{Y(s)}{X(s)} = \underline{\mathbf{\frac{10s^2}{s^2 + 4}}}$$

Chapter 16, Solution 32.

(a) $$Y(s) = H(s)\,X(s)$$

$$= \frac{s+3}{s^2 + 4s + 5} \cdot \frac{1}{s}$$

$$= \frac{s+3}{s(s^2 + 4s + 5)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 4s + 5}$$

$$s + 3 = A(s^2 + 4s + 5) + Bs^2 + Cs$$

Equating coefficients :

s^0 : $3 = 5A \longrightarrow A = 3/5$

s^1: $1 = 4A + C \longrightarrow C = 1 - 4A = -7/5$

s^2 : $0 = A + B \longrightarrow B = -A = -3/5$

$$Y(s) = \frac{3/5}{s} - \frac{1}{5} \cdot \frac{3s + 7}{s^2 + 4s + 5}$$

$$Y(s) = \frac{0.6}{s} - \frac{1}{5} \cdot \frac{3(s+2) + 1}{(s+2)^2 + 1}$$

$$y(t) = \underline{\mathbf{\left[0.6 - 0.6\,e^{-2t}\cos(t) - 0.2\,e^{-2t}\sin(t)\right]u(t)}}$$

(b) $x(t) = 6\,t\,e^{-2t} \longrightarrow X(s) = \dfrac{6}{(s+2)^2}$

$$Y(s) = H(s)\,X(s) = \frac{s+3}{s^2+4s+5}\cdot\frac{6}{(s+2)^2}$$

$$Y(s) = \frac{6(s+3)}{(s+2)^2(s^2+4s+5)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{Cs+D}{s^2+4s+5}$$

Equating coefficients :

s^3: $0 = A + C \longrightarrow C = -A$ (1)

s^2: $0 = 6A + B + 4C + D = 2A + B + D$ (2)

s^1: $6 = 13A + 4B + 4C + 4D = 9A + 4B + 4D$ (3)

s^0: $18 = 10A + 5B + 4D = 2A + B$ (4)

Solving (1), (2), (3), and (4) gives

$A = 6$, $B = 6$, $C = -6$, $D = -18$

$$Y(s) = \frac{6}{s+2} + \frac{6}{(s+2)^2} - \frac{6s+18}{(s+2)^2+1}$$

$$Y(s) = \frac{6}{s+2} + \frac{6}{(s+2)^2} - \frac{6(s+2)}{(s+2)^2+1} - \frac{6}{(s+2)^2+1}$$

$$y(t) = \underline{\left[\mathbf{6\,e^{-2t} + 6\,t\,e^{-2t} - 6\,e^{-2t}\cos(t) - 6\,e^{-2t}\sin(t)}\right]\mathbf{u(t)}}$$

Chapter 16, Solution 33.

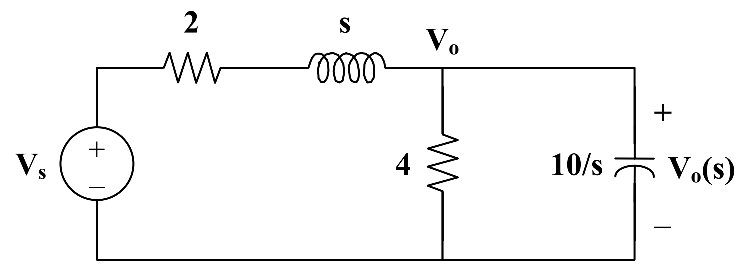
$$H(s) = \frac{Y(s)}{X(s)}, \qquad X(s) = \frac{1}{s}$$

$$Y(s) = \frac{4}{s} + \frac{1}{2(s+3)} - \frac{2s}{(s+2)^2+16} - \frac{(3)(4)}{(s+2)^2+16}$$

$$H(s) = s\,Y(s) = \underline{\mathbf{4 + \frac{s}{2(s+3)} - \frac{2s^2}{s^2+4s+20} - \frac{12s}{s^2+4s+20}}}$$

Chapter 16, Solution 34.

Consider the following circuit.



Using nodal analysis,

$$\frac{V_s - V_o}{s+2} = \frac{V_o}{4} + \frac{V_o}{10/s}$$

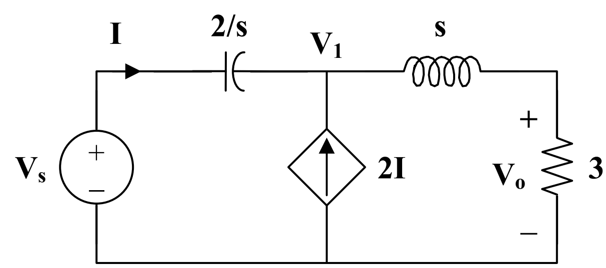
$$V_s = (s+2)\left(\frac{1}{s+2} + \frac{1}{4} + \frac{s}{10}\right)V_o = \left(1 + \frac{1}{4}(s+2) + \frac{1}{10}(s^2+2s)\right)V_o$$

$$V_s = \frac{1}{20}\left(2s^2 + 9s + 30\right)V_o$$

$$\frac{V_o}{V_s} = \underline{\mathbf{\frac{20}{2s^2 + 9s + 30}}}$$

Chapter 16, Solution 35.

Consider the following circuit.



At node 1,

$$2I + I = \frac{V_1}{s+3}, \qquad \text{where} \quad I = \frac{V_s - V_1}{2/s}$$

$$3 \cdot \frac{V_s - V_1}{2/s} = \frac{V_1}{s+3}$$

$$\frac{V_1}{s+3} = \frac{3s}{2} V_s - \frac{3s}{2} V_1$$

$$\left(\frac{1}{s+3} + \frac{3s}{2} \right) V_1 = \frac{3s}{2} V_s$$

$$V_1 = \frac{3s\,(s+3)}{3s^2 + 9s + 2} V_s$$

$$V_o = \frac{3}{s+3} V_1 = \frac{9s}{3s^2 + 9s + 2} V_s$$

$$H(s) = \frac{V_o}{V_s} = \underline{\mathbf{\frac{9s}{3s^2 + 9s + 2}}}$$

Chapter 16, Solution 36.

From the previous problem,

$$3\,I = \frac{V_1}{s+3} = \frac{3s}{3s^2 + 9s + 2} V_s$$

$$I = \frac{s}{3s^2 + 9s + 2} V_s$$

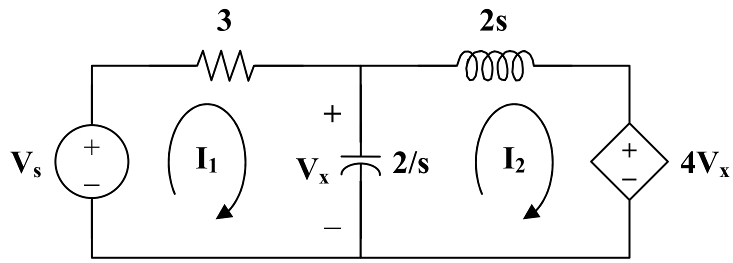
But $\quad V_s = \dfrac{3s^2 + 9s + 2}{9s} V_o$

$$I = \frac{s}{3s^2 + 9s + 2} \cdot \frac{3s^2 + 9s + 2}{9s} V_o = \frac{V_o}{9}$$

$$H(s) = \frac{V_o}{I} = \underline{\mathbf{9}}$$

Chapter 16, Solution 37.

(a) Consider the circuit shown below.



For loop 1,

$$V_s = \left(3 + \frac{2}{s}\right) I_1 - \frac{2}{s} I_2 \qquad (1)$$

For loop 2,

$$4V_x + \left(2s + \frac{2}{s}\right) I_2 - \frac{2}{s} I_1 = 0$$

But, $V_x = (I_1 - I_2)\left(\frac{2}{s}\right)$

So, $\frac{8}{s}(I_1 - I_2) + \left(2s + \frac{2}{s}\right) I_2 - \frac{2}{s} I_1 = 0$

$$0 = \frac{-6}{s} I_1 + \left(\frac{6}{s} - 2s\right) I_2 \qquad (2)$$

In matrix form, (1) and (2) become

$$\begin{bmatrix} V_s \\ 0 \end{bmatrix} = \begin{bmatrix} 3 + 2/s & -2/s \\ -6/s & 6/s - 2s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = \left(3 + \frac{2}{s}\right)\left(\frac{6}{s} - 2s\right) - \left(\frac{6}{s}\right)\left(\frac{2}{s}\right)$$

$$\Delta = \frac{18}{s} - 6s - 4$$

$$\Delta_1 = \left(\frac{6}{s} - 2s\right) V_s, \qquad \Delta_2 = \frac{6}{s} V_s$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{(6/s - 2s)}{18/s - 4 - 6s} V_s$$

$$\frac{I_1}{V_s} = \frac{3/s - s}{9/s - 2 - 3} = \underline{\mathbf{\frac{s^2 - 3}{3s^2 + 2s - 9}}}$$

(b) $$I_2 = \frac{\Delta_2}{\Delta}$$

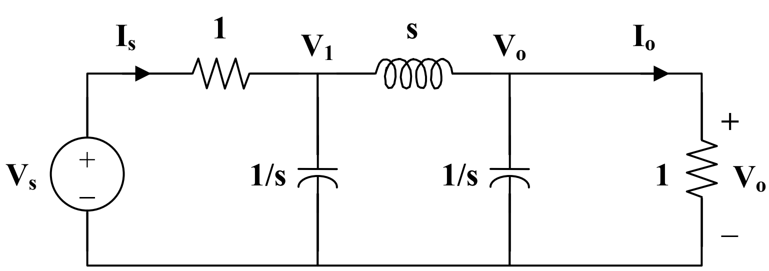
$$V_x = \frac{2}{s}(I_1 - I_2) = \frac{2}{s}\left(\frac{\Delta_1 - \Delta_2}{\Delta}\right)$$

$$V_x = \frac{2/s\, V_s\, (6/s - 2s - 6/s)}{\Delta} = \frac{-4V_s}{\Delta}$$

$$\frac{I_2}{V_x} = \frac{6/s\, V_s}{-4V_s} = \underline{\mathbf{\frac{-3}{2s}}}$$

Chapter 16, Solution 38.

(a) Consider the following circuit.



At node 1,

$$\frac{V_s - V_1}{1} = s V_1 + \frac{V_1 - V_o}{s}$$

$$V_s = \left(1 + s + \frac{1}{s}\right) V_1 - \frac{1}{s} V_o \qquad (1)$$

At node o,

$$\frac{V_1 - V_o}{s} = s\,V_o + V_o = (s+1)\,V_o$$

$$V_1 = (s^2 + s + 1)\,V_o \qquad (2)$$

Substituting (2) into (1)

$$V_s = (s + 1 + 1/s)(s^2 + s + 1)V_o - 1/s\,V_o$$

$$V_s = (s^3 + 2s^2 + 3s + 2)V_o$$

$$H_1(s) = \frac{V_o}{V_s} = \underline{\mathbf{\frac{1}{s^3 + 2s^2 + 3s + 2}}}$$

(b) $I_s = V_s - V_1 = (s^3 + 2s^2 + 3s + 2)V_o - (s^2 + s + 1)V_o$

$I_s = (s^3 + s^2 + 2s + 1)V_o$

$$H_2(s) = \frac{V_o}{I_s} = \underline{\mathbf{\frac{1}{s^3 + s^2 + 2s + 1}}}$$

(c) $I_o = \dfrac{V_o}{1}$

$$H_3(s) = \frac{I_o}{I_s} = \frac{V_o}{I_s} = H_2(s) = \underline{\mathbf{\frac{1}{s^3 + s^2 + 2s + 1}}}$$

(d). $H_4(s) = \dfrac{I_o}{V_s} = \dfrac{V_o}{V_s} = H_1(s) = \underline{\mathbf{\dfrac{1}{s^3 + 2s^2 + 3s + 2}}}$

Chapter 16, Solution 39.

Consider the circuit below.